Escaping Ageing Lions

If you can’t beat em, constrain em.

Who am I?

Josh Vander Hook

w/ Volkan Isler

Robotic Sensor Networks

http://josh.vanderhook.info

http://rsn.cs.umn.edu

josh@vanderhook.info

Why lions and men?

- Pursuit evasion games
- Proposed in 1925 by Rado

- Can show practical limitations
- Can show feasibility of real-world applications

Setup

1. Circular Arena
2. Lion (point)
3. Man (point)
4. Lion wants to capture Man
5. Man wants to keep a separation forever

Varying the problem

- velocity
- kinematics
- information

This talk: “Solutions” to some “interesting” problems, and a brief overview of my current work

Problem 1

Man is at m(t) and lion is at l(t). Is there some finite *t where m(t)==l(t)?*

- Fit lions are fast (vl > vm)
- No constraints on movement

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- Fit lions are fast (vl > vm)
- No constraints on movement

Lion wins, too easy. (Proof?)

Problem 2: Fat lions

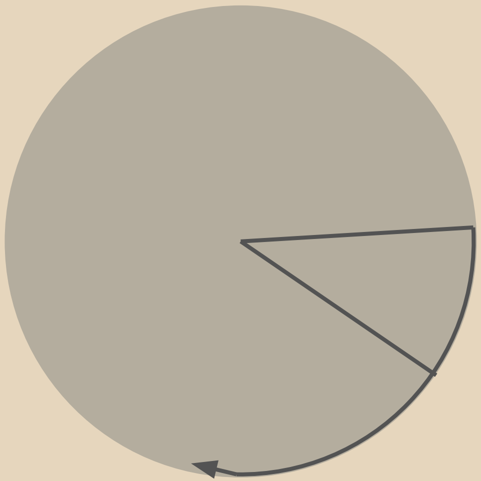
Man is at m(t) and lion is at l(t). Is there some finite *t where m(t)==l(t)?*

- Fat lions are not so fast (vl == vm)
- No other constraints on movement

Not so easy ...

Problem 2: Fat lions

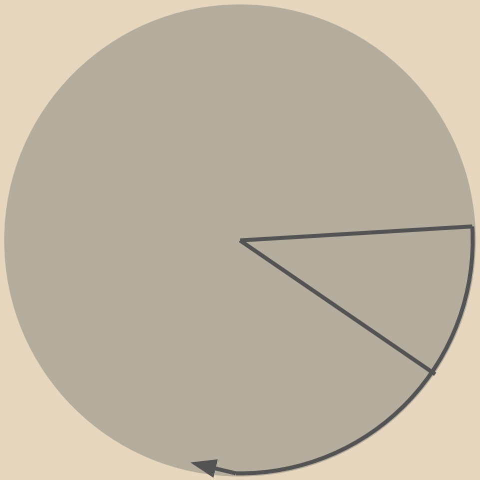
- Run around the outside of the arena
- Lion on “pure pursuit”

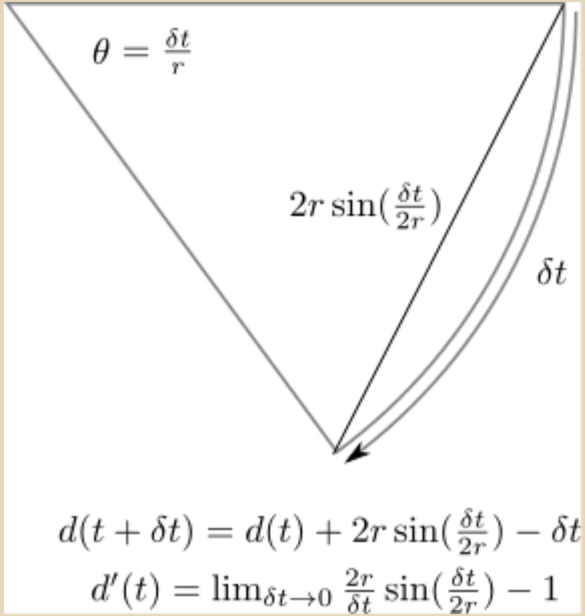


Problem 2: Fat lions

Man escapes! (If lion is doing “pure pursuit”)

- But what about other lion strategies?

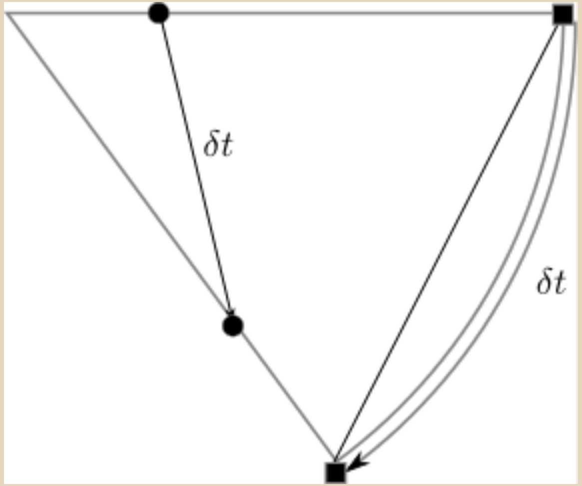




Problem 3: Fat, smarter lions

Lions Counter!

- Introduce the Canonical “lion’s move”
  - (Rado’s own solution)
- Same evader strategy

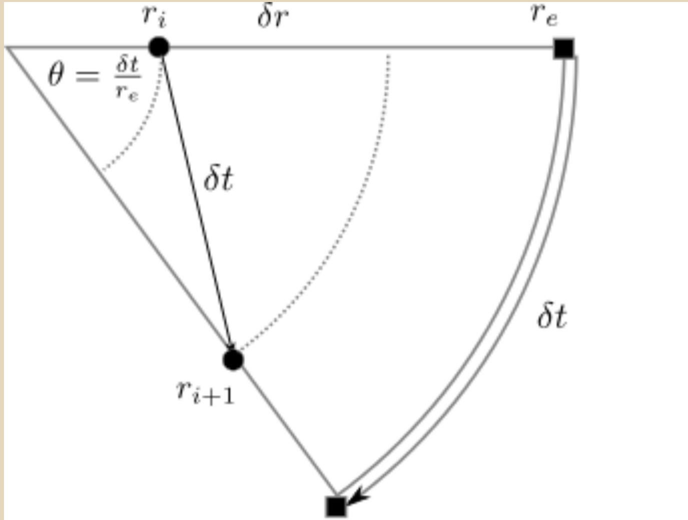


Problem 3: Fatter, smarter lions

Lions Counter!

- Introduce the Canonical “lion’s move”
  - (Rado’s own solution)
- Same evader strategy
- Lion wins again!

- What’s the problem with this counter?



$$\delta t^2 = r_i^2 + r_{i+1}^2 - 2 r_i r_{i+1} \cos\theta$$

$$r' = \lim_{\delta t \to 0} \frac{\sqrt{r^2\left(\cos^2\left(\frac{\delta t}{r_e}\right) - 1\right) + \delta t^2} + r\cos\left(\frac{\delta t}{r_e}\right) - r}{\delta t}$$

$$r' = \sqrt{1 - \frac{r}{r_e}}$$

Problem 3: Fatter, smarter lions

Lions Counter!

- Introduce the Canonical “lion’s move”
    - (Rado’s own solution)
- Same evader strategy
- Lion wins again!

But wait, evader strategy exists to counter *all* continuous-time fatty lion strategies (but lion gets *very close*).

[Besicovitch, 1952] See J. E. Littlewood, Littlewood’s Miscellany (B. Bollob´as, ed.), Cambridge University Press, 1986

Problem 4: Fatter lions with big arms

Man is at $m(t)$ and lion is at $l(t)$ and has a capture radius C. Is there some finite *t where* $|m(t)-l(t)|<C$?

- Equal velocities

Problem 4: Fatter lions with big arms

Man is at m(t) and lion is at l(t) and has a capture radius C. Is there some finite *t where* $|m(t)-l(t)|<C$?

- Equal velocities
- Apply any of the previous continuous time strategies
  - Lion's move most popular
- But how long does it take?
  - Best so far is $O(r/c\ (\log r/c))$
  - 40 years later! [Alonso, Reingold '93]

Problem 5: Superfast lions on bikes

Man is at m(t) and lion is at l(t). Is there some finite *t where m(t)==l(t)?*

- Lions are super fast (vl > vm)
- Lion must have a continuous trajectory
  - with bounded curvature

Not so easy ...

1. Isaacs, 1951
2. Mertz, 1971
3. Lewin, 1973

Problem 6: Superfast lions in wheelchairs

Man is at m(t) and lion is at l(t). Is there some finite *t where m(t)==l(t)?*

- Lions are super fast (vl > vm)
- Lion *uses differential drive kinematics*

Not so easy again ...

1. Ruiz "Time-Optimal Motion Strategies for Capturing an Omnidirectional Evader Using a Differential Drive Robot" TRO 2013

Varying Information

- Can the pursuer *always* see the evader?
- Can the pursuer see *everything* about the evader?

Varying Information

Deer is at $m(t)$ and wolf is at $l(t)$. Is there some finite *D where $|w(t)-d(t)|<D$ for all t?*

- “Cursorial Hunting”
- Wolves aren’t too fast (vw==vd)
- No motion constraints
- Wolves can’t see well
- Open problems!

Example: Position Error

- Game in the plane
- Same Speeds
- Pursuer sees an erroneous *position*, m'(t)
- But $|m'(t)-m(t)|<1$

Can the pursuer maintain the distance to the evader?

- *No, [Gunter Rote, 2003] gave an evader strategy*
- *Distance increases at a rate proportional to* $T^{1/3}$

Does this help the Arena case? Unknown! (I know)

Direction Only : Fat Lions with Cataracts?

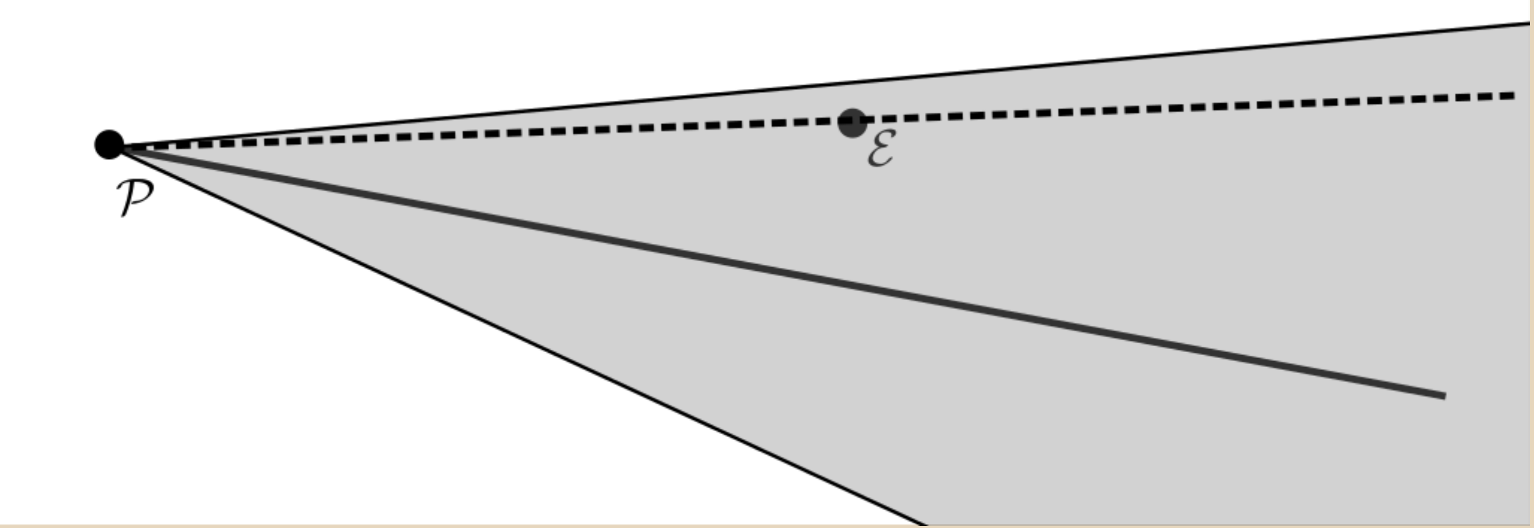
Our recent results!

- Game in the plane
- Same Speeds
- Pursuer sees an erroneous *direction*, b'(t)
- But |b'(t)-b(t)| is bounded.

Can the pursuer maintain the distance to the evader?

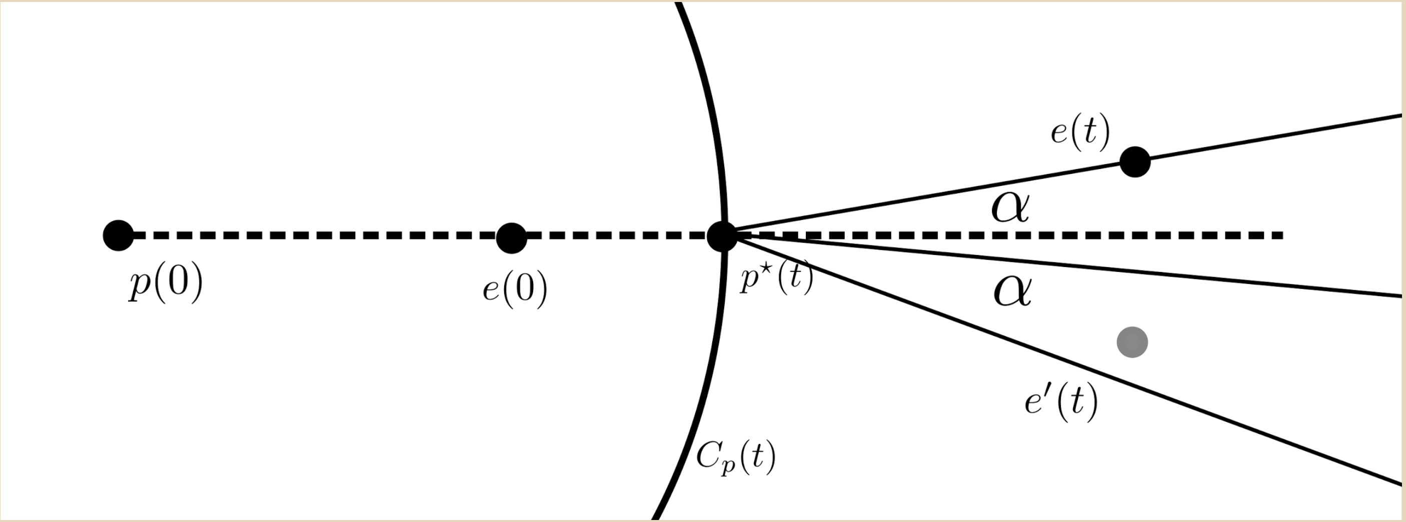
- Special case: b'(t)==b(t)?
- If not, is distance more or less than $T^{1/3}$?

Direction Only : Sensing model



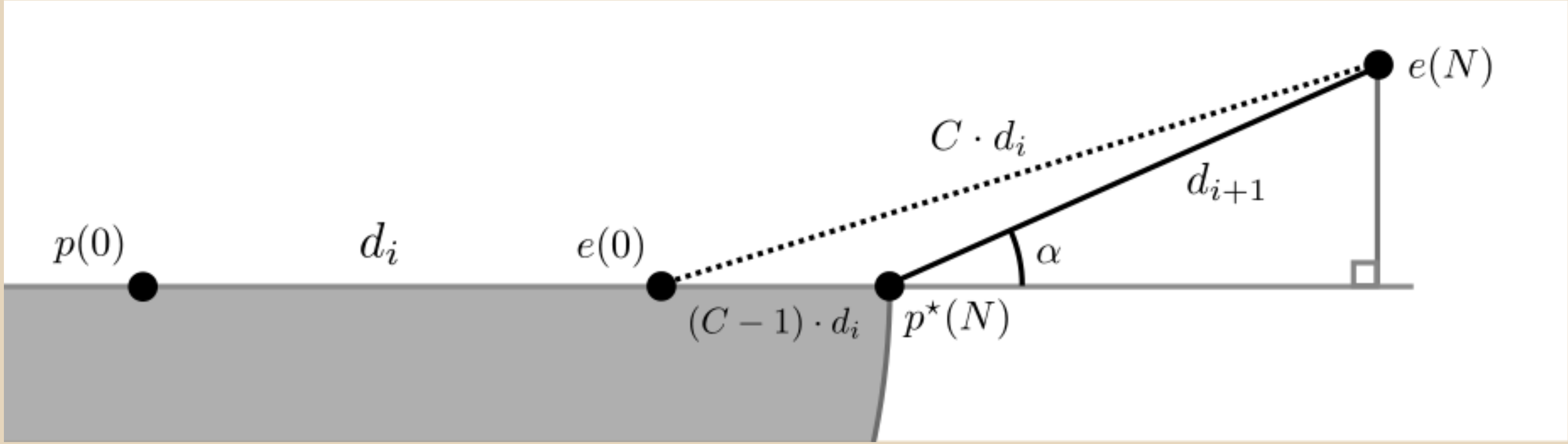
Direction Only : Evader's Strategy

Evader moves along one of two trajectories, e(t) or e'(t), such that *both* are indistinguishable from the pursuer's perspective.

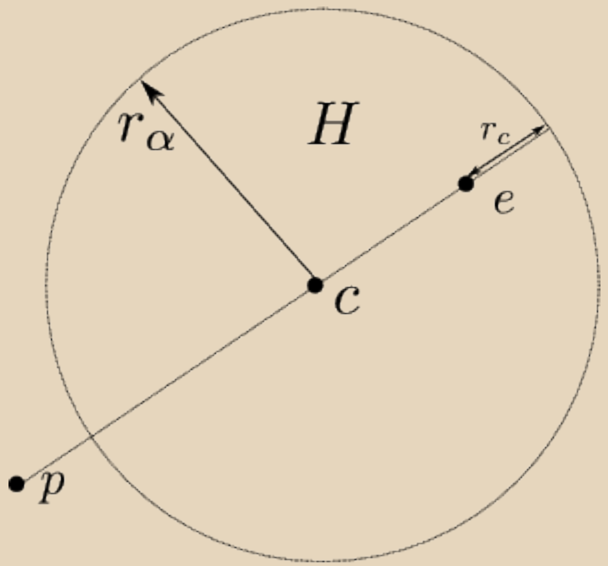


Direction Only : Evader’s Strategy

It turns out, if the evader travels C times d during a round, the ending distance is *more than* the starting distance.



Direction Only : Evader's Strategy



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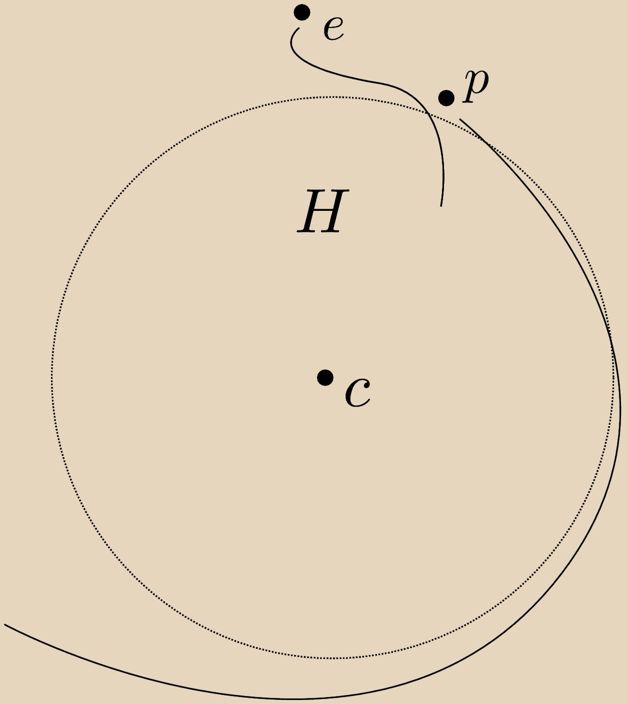
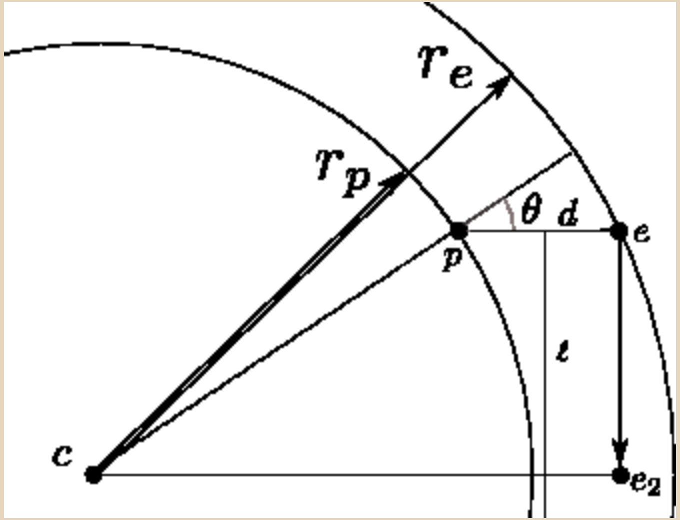
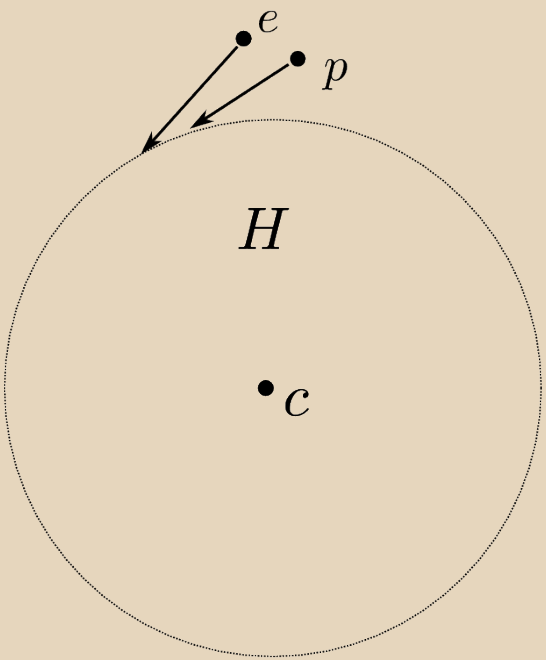


Direction Only : Evader's Strategy





Question

Can we use the distance increase to escape from a lion, even when the arena is bounded?

- Small arenas don’t have enough “room” to execute the distance-increasing strategies
- Current distance-increasing strategies require *straight line trajectories*
- Does a size limit exist for pursuer-win setups?
- If so, does the “quality” of the information affect the size?
- If so, what are the limits on the size as a function of the “quality”

Current work. Answers coming!

Thanks

Josh Vander Hook

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