Escaping Ageing Lions

If you can’t beat em, constrain em.
Who am I?

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Why lions and men?

- Pursuit evasion games
- Proposed in 1925 by Rado
- Can show practical limitations
- Can show feasibility of real-world applications
Varying the problem

- velocity
- kinematics
- information

This talk: “Solutions” to some “interesting” problems, and a brief overview of my current work
Problem 1

*Man is at \( m(t) \) and lion is at \( l(t) \). Is there some finite \( t \) where \( m(t) = l(t) \)?*

- Fit lions are fast (\( v_l > v_m \))
- No constraints on movement
Problem 1

Man is at \( m(t) \) and lion is at \( l(t) \). Is there some finite \( t \) where \( m(t) = l(t) \)?

- Fit lions are fast \((v_l > v_m)\)
- No constraints on movement

Lion wins, too easy. (Proof?)
Problem 2: Fat lions

*Man is at m(t) and lion is at l(t). Is there some finite t where m(t)==l(t)?*

- Fat lions are not so fast (vl == vm)
- No other constraints on movement

Not so easy …
Problem 2: Fat lions

- Run around the outside of the arena
- Lion on “pure pursuit”
Problem 2: Fat lions

Man escapes! (If lion is doing “pure pursuit”)

- But what about other lion strategies?
Problem 3: Fat, smarter lions

Lions Counter!

- Introduce the Canonical “lion’s move”
  - (Rado’s own solution)
- Same evader strategy
Problem 3: Fatter, smarter lions

Lions Counter!

- Introduce the Canonical “lion’s move”
  - (Rado’s own solution)
- Same evader strategy
- Lion wins again!

- What’s the problem with this counter?

\[
\delta t^2 = r_i^2 + r_{i+1}^2 - 2r_i r_{i+1} \cos \theta \\
\theta = \frac{\delta t}{r_e} \\
r' = \lim_{\delta t \to 0} \frac{\sqrt{r^2 (\cos^2 (\frac{\delta t}{r_e}) - 1) + \delta t^2 + r \cos (\frac{\delta t}{r_e})}}{\delta t} \\
r' = \sqrt{1 - \frac{r}{r_e}}
\]
Problem 3: Fatter, smarter lions

Lions Counter!

- Introduce the Canonical “lion’s move”
  - (Rado’s own solution)
- Same evader strategy
- Lion wins again!

But wait, evader strategy exists to counter all continuous-time fatty lion strategies (but lion gets very close).

Problem 4: Fatter lions with big arms

Man is at \( m(t) \) and lion is at \( l(t) \) and has a capture radius \( C \). Is there some finite \( t \) where \( |m(t) - l(t)| < C \)?

- Equal velocities
Problem 4: Fatter lions with big arms

*Man is at* $m(t)$ *and lion is at* $l(t)$ *and has a capture radius* $C$. *Is there some finite* $t$ *where* $|m(t) - l(t)| < C$?

- Equal velocities
- Apply any of the previous continuous time strategies
  - Lion’s move most popular
- But how long does it take?
  - Best so far is $O(r/c \log r/c)$
  - 40 years later! [Alonso, Reingold ‘93]
Problem 5: Superfast lions on bikes

*Man is at* \( m(t) \) *and lion is at* \( l(t) \). *Is there some finite* \( t \) *where* \( m(t) = = l(t) \)?

- Lions are super fast \( (v_l > v_m) \)
- Lion must have a continuous trajectory
  - with bounded curvature

*Not so easy ... *

1. Isaacs, 1951
2. Mertz, 1971
3. Lewin, 1973
Problem 6: Superfast lions in wheelchairs

*Man is at $m(t)$ and lion is at $l(t)$. Is there some finite $t$ where $m(t) == l(t)$?*

- Lions are super fast ($v_l > v_m$)
- Lion uses differential drive kinematics

Not so easy again …

1. Ruiz “Time-Optimal Motion Strategies for Capturing an Omnidirectional Evader Using a Differential Drive Robot” TRO 2013
Varying Information

- Can the pursuer *always* see the evader?
- Can the pursuer see *everything* about the evader?
Deer is at $m(t)$ and wolf is at $l(t)$. Is there some finite $D$ where $|w(t)-d(t)|<D$ for all $t$?

- “Cursorial Hunting”
- Wolves aren’t too fast ($v_w=v_d$)
- No motion constraints
- Wolves can’t see well
- Open problems!
Example: Position Error

- Game in the plane
- Same Speeds
- Pursuer sees an erroneous position, \( m'(t) \)
- But \( |m'(t) - m(t)| < 1 \)

Can the pursuer maintain the distance to the evader?

- No, [Gunter Rote, 2003] gave an evader strategy
- Distance increases at a rate proportional to \( T^{\frac{1}{3}} \)

Does this help the Arena case? Unknown! (I know)
Direction Only : Fat Lions with Cataracts?

Our recent results!
- Game in the plane
- Same Speeds
- Pursuer sees an erroneous direction, $b'(t)$
- But $|b'(t) - b(t)|$ is bounded.

Can the pursuer maintain the distance to the evader?
- Special case: $b'(t) == b(t)$?
- If not, is distance more or less than $T^{\frac{1}{3}}$?
Direction Only: Sensing model
Evader moves along one of two trajectories, \( e(t) \) or \( e'(t) \), such that both are indistinguishable from the pursuer’s perspective.
It turns out, if the evader travels $C$ times $d$ during a round, the ending distance is more than the starting distance.
Direction Only : Evader’s Strategy
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Direction Only: Evader’s Strategy
Can we use the distance increase to escape from a lion, even when the arena is bounded?

- Small arenas don’t have enough “room” to execute the distance-increasing strategies
- Current distance-increasing strategies require *straight line trajectories*
- Does a size limit exist for pursuer-win setups?
- If so, does the “quality” of the information affect the size?
- If so, what are the limits on the size as a function of the “quality”

Current work. Answers coming!
Thanks

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